

# Time-reversal-violating birefringence of photon in a medium exposed to electric and magnetic field.

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## 1 Introduction

Violation of the time reversal symmetry has been observed in  $K_0$ -decay only [1, 2], and remains one of the greatest unsolved problems in the elementary particle physics. A lot of attempts have been undertaken to observe this phenomenon experimentally in different processes. However, those experiments have not been successful.

At the present time novel more precise experiments are actively discussed. Let us note here the experiments with atom (molecule) and neutron spin precession in an electric field  $\vec{E}$  due to interaction of the electric dipole moment (EDM) of atom (molecule) or neutron with the field  $\vec{E}$  [3, 4]. The experiments are discussing for observation of the polarization plane rotation phenomena caused by the pseudo-Zeeman splitting of the atom(molecule) levels by an external electric field  $\vec{E}$  due to interaction  $W = -\vec{d}_a \cdot \vec{E}$  of the atom (molecule) EDM  $\vec{d}_a$  with an electric field [5-9](this effect is similar to the magneto-optic Macaluso-Corbino effect [10]).

It should be noted that the mentioned experiments use the possible existence of such intrinsic quantum characteristic of an atom (molecule) as a static EDM. According to [11, 12, 13] together with the EDM there is one more characteristic of atom (molecule) describing its response to the external field effect - the T- and P-odd polarizability of atom (molecule)  $\beta^T$ . This polarizability differs from zero even if the EDM of an electron is equal to zero and pseudo-Zeeman splitting of atom (molecule) levels is absent.

Both  $\beta^T$  and EDM yield to appearance of several new optical phenomena. Let us mention two of them:

a). the T-odd photon birefringence effect [14] (i.e effect when plane polarized photons are converted to circular polarized ones and vice versa, this effect is similar to magneto-optic birefringence Cotton-Mouton effect [15])

b). the photon polarization plane rotation and circular dichroism in an optically homogeneous isotropic medium exposed to an electric field caused by the Stark mixing of atom (molecule) levels [16, 17]. This T-odd phenomenon is a kinematic analog of the well known T-even phenomenon of Faraday effect of the photon polarization plane rotation in the medium exposed to a magnetic field due to Van-Vleck mechanism. Similarly to the well known P-odd T-even effect of light polarization plane rotation for which the intrinsic spin spiral of atom is responsible [18], this effect is caused by the atom magnetization appearing under external electric field action. Moreover, according to [16], the magnetization of atom appearing under action of static electric field causes the appearance of induced magnetic field  $\vec{H}_{ind}(\vec{E})$ . The energy of interaction of atom magnetic moment  $\vec{\mu}_a$  with this field is  $W_H = -\vec{\mu}_a \cdot \vec{H}_{ind}(\vec{E})$ . Therefore, the total splitting of atom levels is determined by energy  $W_T = -\vec{d}_a \cdot \vec{E} - \vec{\mu}_a \cdot \vec{H}_{ind}(\vec{E})$ . As a result, the effect of polarization plane rotation (birefringence effect) deal with the energy levels splitting is caused not only by  $\vec{d}_a$  interaction

with electric field but by  $\overrightarrow{H}_{ind}(\overrightarrow{E})$  interaction with  $\overrightarrow{\mu}_a$ , too. It is easy to see, that even for  $\overrightarrow{d}_a = 0$  the energy of splitting differs from zero and the T-odd effect of polarization plane rotation and birefringence exist.

In that way the time reversal violating weak interactions yield to the appearance of diverse interesting optical phenomena. In the present paper the optical anisotropy of medium caused by T-odd interactions of electrons and nuclei in atoms and molecules is investigated. The expressions for T-odd polarizabilities of atoms (molecules) in external electric and magnetic fields are obtained. Magnitude of light polarization plane rotation and birefringence effects are estimated. It is shown that investigation of optical anisotropy caused by T-odd interactions can provide information about constants of T-odd weak interactions.

## 2 Tensor of dielectric permittivity of medium at presence of T, P-odd weak interactions.

Let us consider the propagation of electromagnetic wave through the homogeneous isotropic medium placed to the stationary electric  $\overrightarrow{E}$  and magnetic  $\overrightarrow{H}$  fields. Tensor of dielectric permittivity of medium  $\varepsilon_{ik}$  is a certain function of these fields

$$\varepsilon_{ik} = \varepsilon_{ik}(\omega, \overrightarrow{k}, \overrightarrow{E}, \overrightarrow{H}),$$

where  $\omega$  is the frequency and  $\overrightarrow{k}$  is the wave vector of the photon.

Tensor  $\varepsilon_{ik}$  can be presented as the sum

$$\varepsilon_{ik} = \varepsilon_{ik}^{even}(\omega, \overrightarrow{k}, \overrightarrow{E}, \overrightarrow{H}) + \varepsilon_{ik}^{odd}(\omega, \overrightarrow{k}, \overrightarrow{E}, \overrightarrow{H}),$$

where  $\varepsilon_{ik}^{even}(\omega, \overrightarrow{k}, \overrightarrow{E}, \overrightarrow{H})$  is the tensor of dielectric permittivity of medium in the absence of P,T-odd interactions and  $\varepsilon_{ik}^{odd}(\omega, \overrightarrow{k}, \overrightarrow{E}, \overrightarrow{H})$  is the term caused by the T,P-odd weak interactions. Let us expand  $\varepsilon_{ik}$  into irreducible parts:

$$\varepsilon_{ik} = \varepsilon_0 \delta_{ik} + \varepsilon_{ik}^s + \varepsilon_{ik}^a, \quad (1)$$

where  $\delta_{ik}$  is the Kronecker symbol, indices  $i, k = 1, 2, 3$  correspond (x,y,z),  $\varepsilon_0 = \frac{1}{3} \sum_i \varepsilon_{ii}$  is the scalar,  $\varepsilon_{ik}^s = \frac{1}{2}(\varepsilon_{ik} + \varepsilon_{ki}) - \varepsilon_0 \delta_{ik}$  is the symmetric tensor (with trace equal to zero),  $\varepsilon_{ik}^a = \frac{1}{2}(\varepsilon_{ik} - \varepsilon_{ki})$  is the antisymmetric tensor of rank two.

An antisymmetric tensor of rank two can be represented (see, for example [14] as

$$\varepsilon_{ik}^a = i e_{ikl} g_l,$$

where  $\overrightarrow{g}$  is the axled vector dual to the antisymmetric tensor  $\varepsilon_{ik}^a$ ,  $e_{ikl}$  is the fully antisymmetrical unit tensor of rank three.

Tensors  $\varepsilon_{ik}^{even}(\omega, \overrightarrow{k}, \overrightarrow{E}, \overrightarrow{H})$  and  $\varepsilon_{ik}^{odd}(\omega, \overrightarrow{k}, \overrightarrow{E}, \overrightarrow{H})$  can be written by way of (1), too.

Tensor  $\varepsilon_{ik}^{even}(\omega, \overrightarrow{k}, \overrightarrow{E}, \overrightarrow{H})$  describes optical anisotropy of the media in external fields for P,T-even world. Particularly, the symmetrical part of  $\varepsilon_{ik}^{even}$  is responsible for birefringence effects caused by the external fields  $\overrightarrow{E}$  and  $\overrightarrow{H}$  (i.e. effects of Kerr and Cotton-Mouton) and antisymmetrical part describes Faraday and Macaluso-Corbino effects.

Let us consider  $\varepsilon_{ik}^{odd}(\omega, \vec{k}, \vec{E}, \vec{H})$ . Weak interactions are lower than electromagnetic ones. Then, only linear terms in the expansion of  $\varepsilon_{ik}^{odd}$  over weak interaction constant should be taken into consideration. Therefore,  $\varepsilon_{ik}^{odd}$  depends on P,T-odd scalar and tensor combinations of vectors  $\vec{E}$ ,  $\vec{H}$  and  $\vec{k}$  linearly:

$$\begin{aligned}\varepsilon_{ik}^{odd} &= \varepsilon_0^{odd}\delta_{ik} + \varepsilon_{ik}^{(s)odd} + ie_{ikl}g_l^{odd} = \\ &= [\chi_{sEH}^T(\vec{n}_E \vec{n}_H) + \chi_{sE}^T(\vec{n}_\gamma \vec{n}_E) + \chi_{sH}^P(\vec{n}_\gamma \vec{n}_H)]\delta_{ik} + \\ &\quad + \chi_t^T\left[\frac{1}{2}(n_{Ei}n_{Hk} + n_{Hi}n_{Ek}) - \frac{1}{3}(\vec{n}_E \vec{n}_H)\right] + \\ &\quad + i\chi_{sEH}^P e_{ikl}n_{EHi} + i\chi_s^P e_{ikl}n_{\gamma l} + i\chi_E^T e_{ikl}n_{El}\end{aligned}\quad (2)$$

where  $\vec{n}_\gamma = \frac{\vec{k}}{|\vec{k}|}$ ,  $\vec{n}_E = \frac{\vec{E}}{|\vec{E}|}$ ,  $\vec{n}_H = \frac{\vec{H}}{|\vec{H}|}$  and  $\vec{n}_{EH} = \frac{[\vec{E} \times \vec{H}]}{|\vec{E} \times \vec{H}|}$  are the unit vectors, repeated indices imply summation. Term, containing  $\delta_{ik}$ , describes contribution of P,T-odd weak interactions in the scalar part of dielectric polarisability, where  $\chi_{sEH}^T$  determines contribution of T,P-odd interactions,  $\chi_{sE}^T$  is responsible for T-odd, P-even interactions (this contribution was first considered in [19]),  $\chi_{sH}^P$  describes P-odd, T-even interactions. The second term in (2), which is proportional to  $\chi_t^T$ , is responsible for T,P-odd birefringence effect (predicted in [14]). Terms proportional to tensor  $e_{ikl}$  describes light polarisation plane rotation around  $\vec{n}_{EH}$  (P-odd, T-even), around electric field direction  $\vec{n}_E$  (P, T-odd) and around  $\vec{n}_\gamma$  (P-odd, T-even). The latter is well known and actively studied phenomenon of polarisation plane rotation and dichroism caused by P-odd T-even interactions [18, 22, 23]. If spins of atoms (molecules) are certainly oriented then additional contributions to  $\varepsilon_{ik}^{odd}$  can appear [12].

For further consideration let us suppose that medium is optically deluted ( $\varepsilon_{ik} - \delta_{ik} \ll 1$ ). Then the dielectric permittivity tensor  $\varepsilon_{ik}$  depends on the amplitude of elastic coherent scattering of photon by atom (molecule) [11, 12, 13]:

$$\varepsilon_{ik} = \delta_{ik} + \chi_{ik} = \delta_{ik} + \frac{4\pi\rho}{k^2} f_{ik}(0), \quad (3)$$

where  $\chi_{ik}$  is the polarizability tensor of a medium,  $\rho$  is the number of atoms (molecules) per  $cm^3$ ,  $k$  is the photon wave number;  $f_{ik}(0)$  is the tensor part of the zero angle amplitude of elastic coherent scattering of a photon by an atom (molecule). Indices  $i = 1, 2, 3$  are referred to coordinates  $x, y, z$ , respectively.

Similarly  $\varepsilon_{ik}$  tensor  $f_{ik}(0)$  can be expanded in irreducible parts:

$$\begin{aligned}f_{ik}(0) &= f_{ik}^{even} + f_{ik}^{odd} = f_{ik}^{even} + \frac{\omega^2}{c^2} \alpha_{ik}^{odd} = \\ &= f_{ik}^{even} + \frac{\omega^2}{c^2} \{ [\beta_{sEH}^T(\vec{n}_E \vec{n}_H) + \beta_{sE}^T(\vec{n}_\gamma \vec{n}_E) + \beta_{sH}^P(\vec{n}_\gamma \vec{n}_H)]\delta_{ik} + \\ &\quad + \beta_t^T\left[\frac{1}{2}(n_{Ei}n_{Hk} + n_{Hi}n_{Ek}) - \frac{1}{3}(\vec{n}_E \vec{n}_H)\right] + \\ &\quad + i\beta_{sEH}^P e_{ikl}n_{EHi} + i\beta_s^P e_{ikl}n_{\gamma l} + i\beta_E^T e_{ikl}n_{El} \}\end{aligned}\quad (4)$$

where  $\alpha_{ik}^{odd}$  is the tensor of dynamical polarizability of an atom (molecule) and quantities  $\beta$  are the dynamical polarizabilities of an atom (molecule).

The amplitude of elastic coherent scattering of photon by atom (molecule) at zero angle can be written as

$$f(0) = f_{ik}(0)e_i'^* e_k$$

Here  $\vec{e}$  and  $\vec{e}'$  are the unit polarization vectors of initial and scattered photons. The unit vectors describing the circular polarization of photons are:  $\vec{e}'_+ = -\frac{\vec{e}_1 + i\vec{e}_2}{\sqrt{2}}$  for the right and,  $\vec{e}'_- = \frac{\vec{e}_1 - i\vec{e}_2}{\sqrt{2}}$  for the left circular polarization, where  $\vec{e}'_1 \perp \vec{e}'_2$ ,  $\vec{e}'_2 = [\vec{n}'_\gamma \times \vec{e}'_1]$  are the unit polarization vectors of a linearly polarized photon,  $[\vec{e}'_1 \times \vec{e}'_2] = \vec{n}'_\gamma$ ,  $\vec{e}'_1 = -\frac{\vec{e}'_+ - \vec{e}'_-}{\sqrt{2}}$ ,  $\vec{e}'_2 = -\frac{\vec{e}'_+ + \vec{e}'_-}{i\sqrt{2}}$ .

The refractive index is as follows:

$$\hat{N} = N_{ik} = 1 + \frac{2\pi\rho}{k^2} f_{ik}. \quad (5)$$

Suppose  $\vec{H} = 0$  and an electromagnetic wave propagates through a gas along the electric field  $\vec{E}$  direction. The refractive indices for the right  $N_+$  and for the left  $N_-$  circular polarized photons can be written as:

$$N_\pm = 1 + \frac{2\pi\rho}{k^2} f_\pm(0) = 1 + \frac{2\pi\rho}{k^2} \left\{ f^{ev}(0) + \frac{\omega^2}{c^2} \beta_{sE}^T(\vec{n}'_\gamma \vec{n}'_E) \mp \frac{\omega^2}{c^2} [\beta_s^P + \beta_E^T(\vec{n}'_E \vec{n}'_\gamma)] \right\}, \quad (6)$$

where  $f_+(0)(f_-(0))$  is the zero angle amplitude of the elastic coherent scattering of the right (left) circular polarized photon by an atom (molecule).

Let photons with the linear polarization  $\vec{e}'_1 = -\frac{\vec{e}'_+ - \vec{e}'_-}{\sqrt{2}}$  fall in a gas. The polarization vector of a photon in a gas  $\vec{e}'_1$  can be written as:

$$\begin{aligned} \vec{e}'_1 &= -\frac{\vec{e}'_+}{\sqrt{2}} e^{ikN_+L} + \frac{\vec{e}'_-}{\sqrt{2}} e^{ikN_-L} = \\ &= e^{\frac{1}{2}ik(N_+ + N_-)L} \left\{ \vec{e}'_1 \cos \frac{1}{2}k(N_+ - N_-)L - \vec{e}'_2 \sin \frac{1}{2}k(N_+ - N_-)L \right\}, \end{aligned} \quad (7)$$

where  $L$  is the photon propagation length in a medium.

As one can see, the photon polarization plane rotates in a gas. The angle of rotation  $\vartheta$  is

$$\begin{aligned} \vartheta &= \frac{1}{2}k \text{Re}(N_+ - N_-)L = \frac{\pi\rho}{k} \text{Re}[f_+(0) - f_-(0)]L = \\ &= -\frac{2\pi\rho\omega}{c} \text{Re}[\beta_s^P + \beta_E^T(\vec{n}'_E \vec{n}'_\gamma)]L, \end{aligned} \quad (8)$$

where  $\text{Re}N_\pm$  is the real part of  $N_\pm$ . It should be noted that  $\vartheta > 0$  corresponds to the right rotation of the light polarization plane and  $\vartheta < 0$  corresponds to the left one, where the right (positive) rotation is recording by the light observer as the clockwise one.

In accordance with (8) the T-odd interaction results in the photon polarization plane rotation around the electric field  $\vec{E}$  direction. The angle of rotation is proportional to the polarizability  $\beta_E^T$  and the  $(\vec{n}'_E \vec{n}'_\gamma)$  correlation. Together with the T-odd effect there is the well known T-even P-odd polarization plane rotation phenomenon [18, 22, 23] determining by the polarizability  $\beta_s^P$  and being independent on the  $(\vec{n}'_E \vec{n}'_\gamma)$  correlation. The T-odd rotation dependence on the electric field  $\vec{E}$  orientation with respect to the  $\vec{n}'_\gamma$  direction allows one to distinguish T-odd and T-even P-odd phenomena experimentally.

The refractive index  $N_+(N_-)$  has both real and imaginary parts. The imaginary part of the refractive index ( $ImN_{\pm} \sim Im\beta_E^T(\vec{n}_E \vec{n}_\gamma)$ ) is responsible for the T-reversal violating circular dichroism. Due to this process the linearly polarized photon takes circular polarization. The sign of the circular polarization depends on the sign of the scalar production ( $\vec{n}_E \vec{n}_\gamma$ ) that allows us to separate T-odd circular dichroism from P-odd T-even circular dichroism. The last one is proportional to  $Im\beta_s^P$ .

### 3 T, P-odd polarisabilities of atoms and molecules.

In order to estimate the magnitude of the effects one should obtain the T-odd polarizability tensor  $\alpha_{ik}^{odd}$  and T-odd polarizabilities of atom (molecule) or (that is actually the same, see (4,8)) the T-odd part of the amplitude  $f(0)$  of elastic coherent scattering of a photon by an atom (molecule).

According to quantum electrodynamics the elastic coherent scattering at zero angle can be considered as the succession of two processes: the first one is the absorption of the initial photon with the momentum  $\vec{k}$  and the transition of the atom (molecule) from the initial state  $|N_0\rangle$  with the energy  $E_{N_0}$  to an intermediate state  $|F\rangle$  with an energy  $E_F$ ; the second one is the transition of the atom (molecule) from the state  $|F\rangle$  to the final state  $|F'\rangle = |N_0\rangle$  and irradiation of the photon with the momentum  $\vec{k}' = \vec{k}$ .

Let  $H_A$  be the atom (molecule) Hamiltonian considering the weak interaction between electrons and nucleus and the electromagnetic interaction of an atom (molecule) with the external electric  $\vec{E}$  and magnetic  $\vec{H}$  fields. It defines the system of eigenfunctions  $|F\rangle$  and eigenvalues  $E_F = E_F(\vec{E}, \vec{H})$ :

$$H_A |F\rangle = E_F |F\rangle, \quad (9)$$

$F$ -set of quantum numbers describing the state  $|F\rangle$ .

According to [17] T-odd effects in a gas exposed to external electric and magnetic fields are manifested even in electric dipole approximation in contrast to P-odd T-even phenomenon of light polarization plane rotation. Polarizability correspondent to the latter one is proportional to product of matrix elements of electric dipole and magnetic dipole transitions.

The matrix element of the process determining the scattering amplitude in the forward direction in the dipole approximation is given by [20]:

$$\mathfrak{M}_{N_0} = \sum_F \left\{ \frac{\langle N_0 | \vec{d} \vec{e}^* | F \rangle \langle F | \vec{d} \vec{e} | N_0 \rangle}{E_F - E_{N_0} - \hbar\omega} + \frac{\langle N_0 | \vec{d} \vec{e} | F \rangle \langle F | \vec{d} \vec{e}^* | N_0 \rangle}{E_F - E_{N_0} + \hbar\omega} \right\}, \quad (10)$$

where  $\vec{d}$  is the electric dipole transition operator,  $\omega$  is the photon frequency,  $|N_0\rangle$  is the wave function of the initial state of atom (molecule) with the energy  $E_{N_0}$ ,  $|F\rangle$  is the wave function of an intermediate state with an energy  $E_F$ .

For gases the energy of atom  $E_F$  is composed from the internal energy of atom, its kinetic energy and energy of atom interaction with external fields. It is very important to note that in addition to interaction of atom with fields  $\vec{E}$  and  $\vec{H}$  it undergoes influence caused by coherent scattering of an atom by other atoms of gas. Potential energy of this interaction [11, 12] is

$$U_F = -\frac{4\pi\hbar^2}{M_A} \rho f_F(\vec{\mathcal{A}}, \vec{\mathcal{A}}),$$

where  $\rho$  is the density of atoms of gas,  $M_A$  is the mass of atom,  $f_F(\vec{\alpha}, \vec{\alpha})$  is the amplitude of forward elastic coherent scattering of the atom being in the state  $|F\rangle$  by atom of gas (if atoms are identical then exchange scattering also contributes in this amplitude). Energy  $U_F$  depends on the atom state  $|F\rangle$  and orientation of the total moment of an atom in this state. Thus it is fundamentally important to consider  $U_F$  in detail calculation because it can contribute in the effects discussed.

Motion of gas atoms yields to Doppler shift of levels and in order to get the final expressions the equation (8) should be averaged over atom momenta distribution in gas. Hereinafter we will not draw explicitly this routine procedure [18]

It should be reminded that the atom (molecule) excited levels are quasistationary i.e.  $E_F$  has the imaginary part and everywhere  $E_F$  should be presented as  $(E_F - \frac{i}{2}\Gamma_F)$ , where  $E_F$  is the atom (molecule) level energy,  $\Gamma_F$  is the level width.

The matrix element (10) can be written as:

$$\mathfrak{M}_{N_0} = \alpha_{ik}^{N_0} e_i^* e_k, \quad (11)$$

where  $\alpha_{ik}^{N_0}$  is the tensor of dynamical polarizability of an atom (molecule)

$$\alpha_{ik}^{N_0} = \sum_F \left\{ \frac{\langle N_0 | d_i | F \rangle \langle F | d_k | N_0 \rangle}{E_F - E_{N_0} - \hbar\omega} + \frac{\langle N_0 | d_k | F \rangle \langle F | d_i | N_0 \rangle}{E_F - E_{N_0} + \hbar\omega} \right\} \quad (12)$$

In general case atoms are distributed to the levels of ground state  $N_0$  with the probability  $P(N_0)$ . Therefore,  $\alpha_{ik}^{N_0}$  should be averaged over all states  $N_0$ . As a result, the polarizability can be written

$$\alpha_{ik} = \sum_{N_0} P(N_0) \alpha_{ik}^{N_0} \quad (13)$$

The tensor  $\alpha_{ik}$  can be expanded in the irreducible parts as

$$\alpha_{ik} = \alpha_0 \delta_{ik} + \alpha_{ik}^s + \alpha_{ik}^a, \quad (14)$$

where  $\alpha_0 = \frac{1}{3} \sum_i \alpha_{ii}$  is the scalar,  $\alpha_{ik}^s = \frac{1}{2}(\alpha_{ik} + \alpha_{ki}) - \alpha_0 \delta_{ik}$  is the symmetric tensor of rank two,  $\alpha_{ik}^a = \frac{1}{2}(\alpha_{ik} - \alpha_{ki})$  is the antisymmetric tensor of rank two,

$$\begin{aligned} \alpha_0 &= \frac{2}{3} \sum_{N_0} P(N_0) \sum_{iF} \frac{\omega_{N_0} \langle N_0 | d_i | F \rangle \langle F | d_i | N_0 \rangle}{\hbar(\omega_{FN_0}^2 - \omega^2)} \\ \alpha_{ik}^s &= \sum_{N_0} P(N_0) \sum_F \frac{\omega_{N_0} [\langle N_0 | d_i | F \rangle \langle F | d_k | N_0 \rangle + \langle N_0 | d_k | F \rangle \langle F | d_i | N_0 \rangle]}{\hbar(\omega_{FN_0}^2 - \omega^2)} - \alpha_0 \delta_{ik} \quad (15) \\ \alpha_{ik}^a &= \frac{\omega}{\hbar} \sum_{N_0} P(N_0) \sum_F \frac{\langle N_0 | d_i | F \rangle \langle F | d_k | N_0 \rangle - \langle N_0 | d_k | F \rangle \langle F | d_i | N_0 \rangle}{\omega_{FN_0}^2 - \omega^2} \end{aligned}$$

where  $\omega_{FN_0} = \frac{E_F - E_{N_0}}{\hbar}$ .

Let atoms (molecules) be nonpolarized. The antisymmetric part of polarizability  $\alpha_{ik}^a$  is equal to zero in the absence of T- and P- odd interactions. It should be reminded that according to the above for the P-odd and T-even interactions the antisymmetric part of polarizability differs from zero only while considering both the electric and magnetic dipole transitions [18].

As it was shown above, scalar, symmetric and antisymmetric parts of  $\epsilon_{ik}$  (and, therefore,  $\alpha_{ik}$ ) describes some new T- and P-odd effects. For example, the effect of polarisation plane rotation is described by the antisymmetric part  $\alpha_{ik}^a$  of tensor of dynamical polarizability of atom (molecule)  $\alpha_{ik}$ . We can evaluate  $\alpha_{ik}^a$  and, as a result, obtain the expression for  $\beta_E^T$  by the following way. According to (4,6) the magnitude of the T-odd effect is determined by the polarizability  $\beta_E^T$  or (that is actually the same, see (8)) by the amplitude  $f_{\pm}(0)$  of elastic coherent scattering of a photon by an atom (molecule). If  $\vec{n}_E \parallel \vec{n}_\gamma$  the amplitude  $f_{\pm}(0)$  in the dipole approximation can be written as  $f_{\pm} = \mp \frac{\omega^2}{c^2} \beta_E^T$ . As a result, in order to obtain the amplitude  $f_{\pm}$ , the matrix element (10,11) for photon polarization states  $\vec{e} = \vec{e}_{\pm}$  should be found.

The electric dipole transition operator  $\vec{d}$  can be written in the form:

$$\vec{d} = d_+ \vec{e}_+ + d_- \vec{e}_- + d_z \vec{n}_\gamma, \quad (16)$$

with  $\vec{d}_+ = -\frac{d_x - id_y}{\sqrt{2}}$ ,  $\vec{d}_- = \frac{d_x + id_y}{\sqrt{2}}$ . Let photon polarization state  $\vec{e} = \vec{e}_+$ . Using (10,11) we can present the polarizability  $\beta_E^T$  as follows:

$$\beta_E^T = \frac{\omega}{\hbar} \sum_{N_0} P(N_0) \sum_F \left\{ \frac{\langle N_0 | d_- | F \rangle \langle F | d_+ | N_0 \rangle - \langle N_0 | d_+ | F \rangle \langle F | d_- | N_0 \rangle}{\omega_{FN_0}^2 - \omega^2} \right\}. \quad (17)$$

For further analysis the more detailed expressions for atom (molecule) wave functions are necessary. The weak interaction constant is very small. Therefore, we can use the perturbation theory. Let  $|f, E\rangle$  be the wave function of an atom (molecule) interacting with an electric field  $\vec{E}$  in the absence of weak interaction. Switch on weak interaction ( $V_w \neq 0$ ). According to the perturbation theory [20] the wave function  $|F\rangle$  can be written in this case as:

$$|F\rangle = |f, \vec{E}\rangle + \sum_n \frac{\langle n, \vec{E} | V_w | f, \vec{E} \rangle}{E_f - E_n} |n, \vec{E}\rangle \quad (18)$$

It should be mentioned that both numerator and denominator of (17) contain  $V_w$ . Suppose  $V_w$  to be small one can represent the total polarizability  $\beta_E^T$  as the sum of two terms

$$\beta_E^T = \beta_{mix}^T + \beta_{split}^T, \quad (19)$$

where

$$\beta_{mix}^T = \frac{\omega}{\hbar} \sum_{N_0} P(N_0) \sum_f \frac{1}{\omega_{fN_0}^2 - \omega^2} \sum_l \quad (20)$$

$$\left\{ \frac{2Re[\{\langle n_0 \vec{E} | d_- | f \vec{E} \rangle \langle f \vec{E} | d_+ | l \vec{E} \rangle - \langle n_0 \vec{E} | d_+ | f \vec{E} \rangle \langle f \vec{E} | d_- | l \vec{E} \rangle\} \langle l \vec{E} | V_w | n_0 \vec{E} \rangle]}{E_{n_0} - E_l} + \right. \\ \left. + \frac{2Re[\langle n_0 \vec{E} | d_- | l \vec{E} \rangle \langle l \vec{E} | V_w | f \vec{E} \rangle \langle f \vec{E} | d_+ | n_0 \vec{E} \rangle - \langle n_0 \vec{E} | d_+ | l \vec{E} \rangle \langle l \vec{E} | V_w | f \vec{E} \rangle \langle f \vec{E} | d_- | n_0 \vec{E} \rangle]}{E_f - E_l} \right\}$$

and

$$\begin{aligned}
\beta_{split}^T &= \frac{\omega}{\hbar} \sum_{N_0} P(N_0) \sum_F \left\{ \frac{\langle n_0 | d_- | f \rangle \langle f | d_+ | n_0 \rangle - \langle n_0 | d_+ | f \rangle \langle f | d_- | n_0 \rangle}{\omega_{FN_0}^2 - \omega^2} \right\} = \\
&= \frac{\omega}{\hbar} \sum_{N_0} P(N_0) \sum_F \left\{ \frac{\langle n_0 | d_- | f \rangle \langle f | d_+ | n_0 \rangle - \langle n_0 | d_+ | f \rangle \langle f | d_- | n_0 \rangle}{(\omega_{FN_0} - \omega)(\omega_{FN_0} + \omega)} \right\} = \\
&= \frac{1}{2\hbar} \sum_{N_0} P(N_0) \sum_F \left\{ \frac{\langle n_0 | d_- | f \rangle \langle f | d_+ | n_0 \rangle - \langle n_0 | d_+ | f \rangle \langle f | d_- | n_0 \rangle}{(\omega_{FN_0} - \omega)} \right\} \quad (21)
\end{aligned}$$

$$\omega_{FN_0} = \frac{E_F(\vec{E}) - E_{N_0}(\vec{E})}{\hbar},$$

It should be reminded that according to all the above (see also section 3) energy levels  $E_F$  and  $E_{N_0}$  contain shifts caused by interaction of electric dipole moment of the level with electric field  $\vec{E}$  and magnetic moment of the level with T-odd induced magnetic field  $\vec{H}_{ind}(\vec{E})$ .

It should be noted that radial parts of the atom wave functions are real [20, 21], therefore the matrix elements of operators  $d_{\pm}$  are real too. As a result, the P-odd T-even part of the interaction  $V_w$  does not contribute to  $\beta_{mix}^T$  because the P-odd T-even matrix elements of  $V_w$  are imaginary [18]. At the same time, the T- and P-odd matrix elements of  $V_w$  are real [18], therefore, the polarizability  $\beta_{mix}^T \neq 0$ . It should be mentioned that in the absence of electric field ( $\vec{E} = 0$ ) the polarizability  $\beta_E^T = 0$  and, therefore, the phenomenon of the photon polarization plane rotation is absent.

Really, the electric field  $\vec{E}$  mixes the opposite parity levels of the atom. The atom levels have the fixed parity at  $\vec{E} = 0$ . The operators  $d_{\pm}$  and  $V_w$  change the parity of the atom states. As a result, the parity of the final state  $|N'_0\rangle = d_+ d_- V_w |N_0\rangle$  appears to be opposite to the parity of the initial state  $|N_0\rangle$ . But the initial and final states in the expression for  $\beta_E^T$  are the same. Therefore  $\beta_E^T$  can not differ from zero at  $\vec{E} = 0$ .

It should be emphasized once again that polarizability  $\beta_E^T$  differs from zero even if EDM of electron is equal to zero. The interaction of electron EDM with electric field gives only part of contribution to the total polarizability of atom (molecule). The new effect we discuss is caused by the Stark mixing of atom (molecule) levels and weak T- and P-odd interaction of electrons with nucleus (and with each other).

Therefore, according to (19) the total angle of polarization plane rotation includes two terms  $\vartheta = \vartheta_{mix} + \vartheta_{split}$ , where  $\vartheta_{mix} \sim \beta_{mix}^T$  is caused by the considered above effect similar to Van Vleck that and  $\vartheta_{split} \sim \beta_{split}^T$  is caused by the atom levels splitting both in electric field  $\vec{E}$  and magnetic field  $\vec{H}_{ind}(\vec{E})$ . The contributions given by  $\beta_{mix}^T$  and  $\beta_{split}^T$  can be distinguished by studying the frequency dependence of  $\vartheta = \vartheta(\omega)$ . According to (20,21)  $\vartheta_{mix} \sim \frac{1}{\omega_{fn_0} - \omega}$  whereas  $\vartheta_{split} \sim \frac{1}{(\omega_{fn_0} - \omega)^2}$ . So,  $\vartheta_{split}$  decreases faster than  $\vartheta_{mix}$  with the grows of frequency tuning out from resonance.

Let us now estimate the magnitude of the effect of the T-odd photon plane rotation due to  $\beta_{mix}^T$ . According to the analysis [11, 12, 13], based on the calculations of the value of T- and P-noninvariant interactions given by [18], the ratio  $\frac{\langle V_w^T \rangle}{\langle V_w^P \rangle} \leq 10^{-3} \div 10^{-4}$ , where  $\langle V_w^T \rangle$  is T and P-odd matrix element,  $\langle V_w^P \rangle$  is P-odd T-even matrix element.

The P-odd T-even polarizability  $\beta_s^P$  is proportional to the product of electric and magnetic dipole matrix elements and  $\langle V_w^P \rangle$ :  $\beta_s^P \sim \langle d \rangle \langle \mu \rangle \langle V_w^P \rangle$  [18]. At the same time  $\beta_{mix}^T \sim$



$\langle d(\vec{E}) \rangle \langle d(\vec{E}) \rangle \langle V_w^T \rangle$ . As a result,

$$\frac{\beta_{mix}^T}{\beta_s^P} \sim \frac{\langle d(\vec{E}) \rangle \langle d(\vec{E}) \rangle \langle V_w^T \rangle}{\langle d \rangle \langle \mu \rangle \langle V_w^P \rangle}. \quad (22)$$

Let us study the T-odd phenomena of the photon polarization plane rotation in an electric field  $\vec{E}$  for the transition  $n_0 \rightarrow f$  between the levels  $n_0$  and  $f$  which have the same parity at  $\vec{E} = 0$ . The matrix element  $\langle n_0, \vec{E} | d_{\pm} | f, \vec{E} \rangle$  does not equal to zero only if  $\vec{E} \neq 0$ . Let the energy of interaction of an atom with an electric field,  $V_E = -\vec{d} \cdot \vec{E}$ , be much smaller than the spacing  $\Delta$  of the energy levels, which are mixed by the field  $\vec{E}$ . Then one can use the perturbation theory for the wave functions  $|f, \vec{E}\rangle$ :

$$|f, \vec{E}\rangle = |f\rangle + \sum_m \frac{\langle m | -d_z E | f \rangle}{E_f - E_m} |m\rangle, \quad (23)$$

where  $z \parallel \vec{E}$ . As a result, the matrix element  $\langle n_0, \vec{E} | d_{\pm} | f, \vec{E} \rangle$  can be rewritten as:

$$\begin{aligned} \langle n_0, \vec{E} | d_{\pm} | f, \vec{E} \rangle &= \\ &= - \left\{ \sum_m \frac{\langle n_0 | d_{\pm} | m \rangle \langle m | d_z | f \rangle}{E_f - E_m} + \right. \\ &\quad \left. + \sum_p \frac{\langle n_0 | d_z | p \rangle \langle p | d_{\pm} | f \rangle}{E_{n_0} - E_p} \right\} E. \end{aligned} \quad (24)$$

One can see that the matrix element  $\langle d(\vec{E}) \rangle \sim \frac{\langle dE \rangle}{\Delta} \langle d \rangle$  in this case. The other matrix elements in (20) can be evaluated at  $\vec{E} = 0$ . This gives the estimate as follows:

$$\beta_{mix}^T \sim \langle d \rangle \langle d \rangle \frac{\langle dE \rangle}{\Delta} \langle V_w^T \rangle. \quad (25)$$

and, consequently, ratio (22) can be written as

$$\frac{\beta_{mix}^T}{\beta_s^P} \sim \frac{\langle d \rangle \langle d \rangle \frac{\langle dE \rangle}{\Delta} \langle V_w^T \rangle}{\langle d \rangle \langle \mu \rangle \langle V_w^P \rangle}. \quad (26)$$

Taking into account that the matrix element  $\langle \mu \rangle \sim \alpha \langle d \rangle$  [20, 21], where  $\alpha = \frac{1}{137}$  is the fine structure constant, equation (26) can be reduced to:

$$\frac{\beta_{mix}^T}{\beta_s^P} \sim \alpha^{-1} \frac{\langle dE \rangle}{\Delta} \frac{\langle V_w^T \rangle}{\langle V_w^P \rangle} \quad (27)$$

For the case  $\frac{\langle dE \rangle}{\Delta} \sim 1$ , ratio (27) gives

$$\frac{\beta_{mix}^T}{\beta_s^P} \sim \alpha^{-1} \frac{\langle V_w^T \rangle}{\langle V_w^P \rangle} \lesssim 10^{-1} \div 10^{-2} \quad (28)$$

Such condition can be realized, for example, for excited states of atoms and for two-atom molecules (TlF, BiS, HgF) which have a pair of nearly degenerate opposite parity states. As one can see, the ratio  $\frac{\beta_{mix}^T}{\beta_s^P}$  is two orders larger as compared with the simple estimation  $\frac{\langle V_w^T \rangle}{\langle V_w^P \rangle} \leq 10^{-3} \div 10^{-4}$  due to the fact that  $\beta_{mix}^T$  is determined by only the electric dipole transitions, while  $\beta_s^P$  is determined by both the electric and magnetic dipole transitions.

## 4 The possibility to observe the time-reversal violating optical phenomena experimentally.

The possibility to observe the time-reversal violating optical phenomena experimentally can be discussed now. In accordance with (6) the angle of the T-odd rotation in electric field can be evaluated as follows:

$$\vartheta_{mix}^T \sim \frac{2\pi\rho\omega}{c}\beta_{mix}^T L \sim \frac{\beta_{mix}^T}{\beta_S^P}\vartheta^P \sim \alpha^{-1} \frac{\langle dE \rangle}{\Delta} \frac{\langle V_w^T \rangle}{\langle V_w^P \rangle} \vartheta^P. \quad (29)$$

According to the experimental data [22, 23] being well consistent with calculations [18] the typical value of  $\vartheta^P$  is  $\vartheta^P \sim 10^{-6} rad$  (for the length  $L$  being equal to the several absorption lengths of the light propagating through a gas  $L_a$ ).

For the electric field  $E \sim 10^4 V \cdot cm^{-1}$  the parameter  $\frac{\langle dE \rangle}{\Delta}$  can be estimated as  $\frac{\langle dE \rangle}{\Delta} \sim 10^{-5}$  for Cs, Tl and  $\frac{\langle dE \rangle}{\Delta} \sim 10^{-4}$  for Yb and lead. Therefore, one can obtain  $\vartheta_{mix}^T \sim 10^{-13} rad$  for Cs, Tl and  $\vartheta_{mix}^T \sim 10^{-12} rad$  for Yb and lead. For the two-atom molecules (TlF, BiS, HgF) the angle  $\vartheta_{mix}^T$  can be larger, because they have a pair of degenerate opposite parity states.

It should be noted that the classical up-to-date experimental techniques allow to measure angles of light polarization plane rotation up to  $4,3 \cdot 10^{-11} rad$  [24].

A way to increase the rotation angle  $\vartheta^T$  is to increase the length  $L$  of the path of a photon inside a medium (see (6)). It can be done, for example, by placing a medium in a resonator or inside a laser gyroscope. This becomes possible due to the fact that in contrast with the phenomenon of P-odd rotation of the polarization plane of photon the T-odd rotation in an electric field is accumulated while photon is moving both in the forward and backward directions.

For the first view the re-reflection of the wave in resonator can not provide the significant increase of the photon path length  $L$  in comparison with the absorption length  $L_a$  because of the absorption of photons in a medium. Nevertheless this difficulty can be overcome when the part of resonator is filled by the amplifying medium (for example, inverse medium). As a result, the electromagnetic wave being absorbed by the investigated gas is coherently amplified in the amplifier and then is refracted to the gas again. Consequently, under the ideal conditions the light pulse can exist in such resonator-amplifier for arbitrarily long time. And the peculiar "photon trap", in which photon polarisation plane rotates, appears (Fig.1). The angle of rotation  $\vartheta_t^T = \Omega^T \cdot t$ , where  $\Omega^T$  is the frequency of the photon polarization plane rotation around the  $\vec{E}$  direction,  $t$  is the time of electromagnetic wave being in a "trap". It is easy to find the frequency  $\Omega^T$  from (6):  $\Omega^T = \frac{\vartheta^T}{L}c = 2\pi\rho\omega\beta_E^T$ . From the estimates of  $\vartheta^T$  it is evident that for  $\vartheta^T \sim 10^{-12} rad$  (Lead, Yb) the frequency  $\Omega^T$  appears to be  $\Omega^T = \frac{\vartheta^T}{L_a}c \sim 10^{-4} sec^{-1}$ . Therefore  $\vartheta_t^T \sim 10^{-4}t$  and for the time  $t$  of about 3 hours the angle  $\vartheta_t^T$  becomes  $\sim 1 rad$ . The similar estimates for the atoms Cs, Tl ( $\vartheta^T \sim 10^{-13} rad$ ) give that for the same time the angle  $\vartheta_t^T \sim 10^{-1} rad$ .

The time  $t$  is limited, in particular, by spontaneous radiation of photons in an amplifier that gradually leads to the depolarization of photon gas in resonator. Surely, it is the ideal picture, but here is the way to further increase of the experiment sensitivity. The achieved sensitivity in measurements of phase incursion in laser gyroscope makes possible to observe the effect in laser gyroscope, too. Laser interferometers used as gravitational wave detectors also can provide necessary sensitivity.

Requiring to measure rotation angle  $\sim 10^{-6} rad$  in "photon trap" and taking into consideration that existing technique allows to measure much less angles one can expect to observe effect of the order  $\frac{V_T}{V_P} \sim 10^{-9} \div 10^{-10}$ .

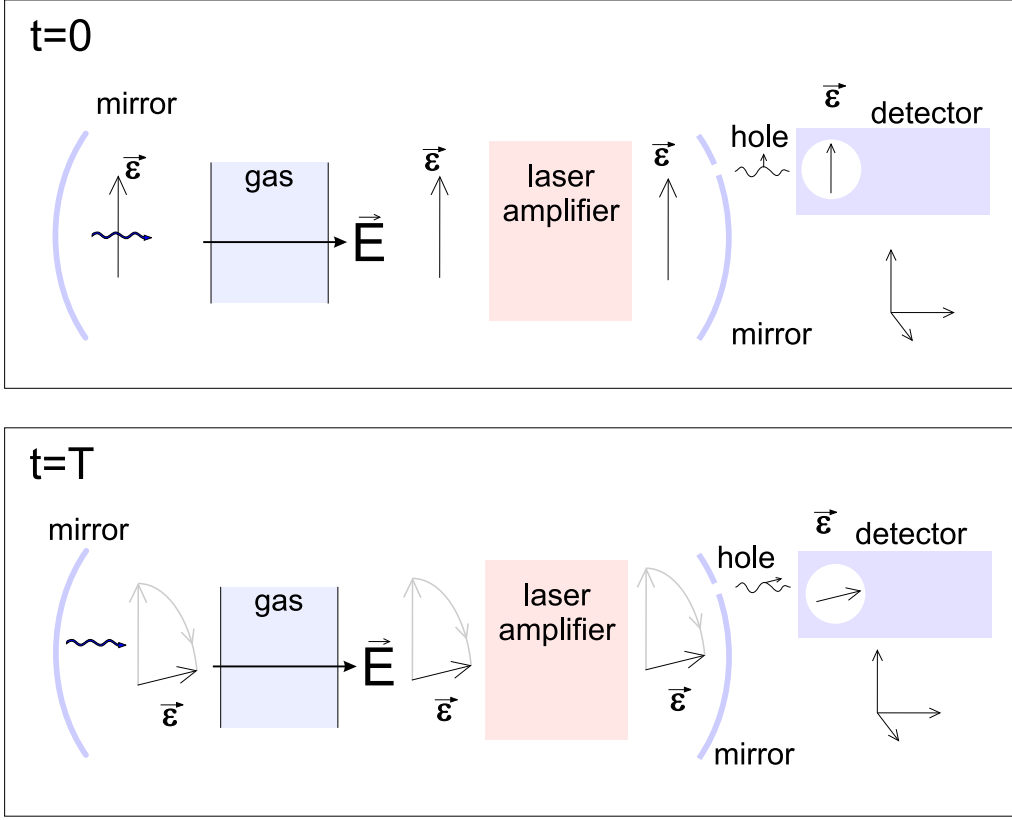


Figure 1: Polarization plane of photons in a trap rotates in time around the electric field direction.

All the estimations discussed above for the photon polarisation plane rotation phenomena can be fully referred to birefringence effect which is described by the symmetrical part of T-odd polarisability tensor. Two effects contribute both in polarisation plane rotation and birefringence phenomena. They are:

1. splitting of atom (molecule) levels
2. mixing of ground state and opposite parity states in external fields

Birefringence effect appears when photon moves orthogonally to the electric and magnetic fields. The magnitude of effect is proportional to the correlation  $(\vec{E} \vec{H})$ . Thus, one can distinguish the T-odd birefringence effect against a background of T-even one by changing the direction of  $\vec{E}$  with respect to direction of  $\vec{H}$ .

For birefringence effect it should be mentioned that change of polarisation type occurs in "photon trap" in time: circular polarisation is converted into linear one, then linear polarisation is converted into circular one and so on (see Fig. (2))

Suppose effects of polarization plane rotation and birefringence be caused only by atom EDM one can estimate the possible sensitivity of EDM measurement in such experiments. Suppose we will measure rotation angle with sensitivity about  $10^{-6} \text{ rad/hour}$  (degree of circular polarisation in the birefringence effect  $\delta \sim 10^{-6} \text{ per/hour}$ ). Rotation angle is  $\delta\vartheta = k \text{Re}(N_+ - N_-)L = k \text{Re}(N_+ - N_-)cT$ , where  $T$  is the observation time (degree of circular polarisation at conversion from linear polarisation to circular that  $\delta = k \text{Re}(N_{\parallel} - N_{\perp})L = k \text{Re}(N_{\parallel} - N_{\perp})cT$ ), here  $N_{\parallel}$  is the index of refraction of photon with linear polarization parallel to the electric field and  $N_{\perp}$  is that perpendicular to the electric field. Representing

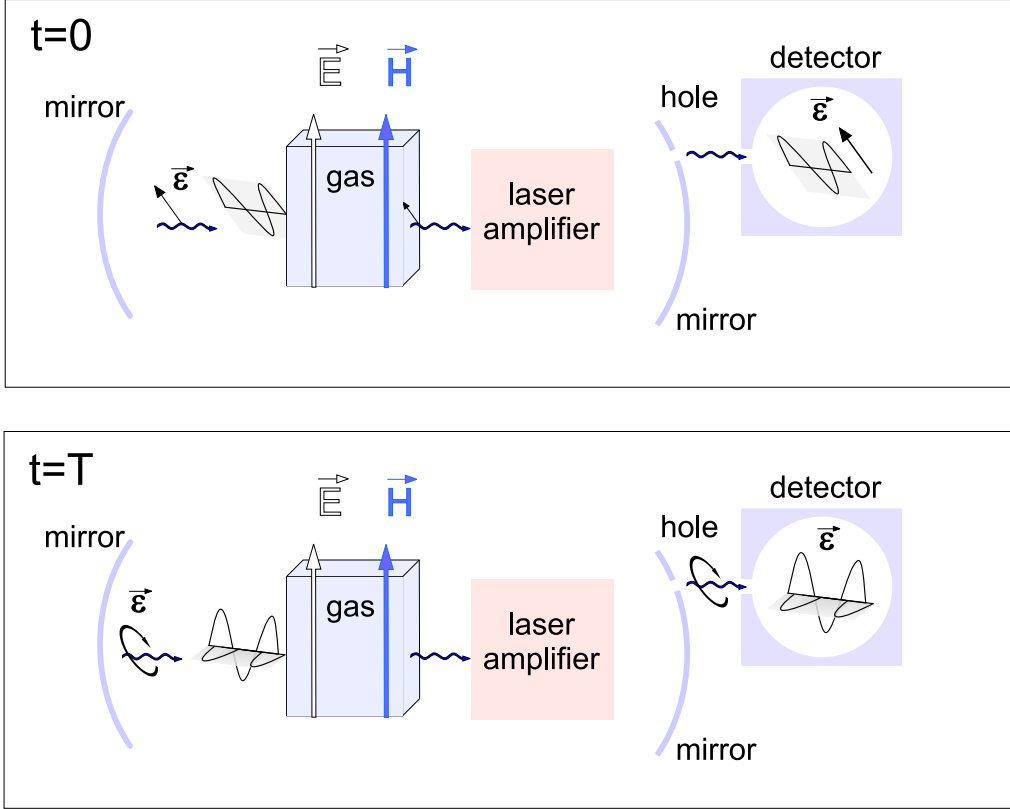


Figure 2: Owing to birefringence effect in "photon trap" linear polarisation is converted into circular one and vice versa

$\delta\vartheta$  in the form

$$\delta\vartheta = \frac{\rho c T \lambda^2 \Gamma_e d_a E}{2\pi \hbar \Gamma^2} \quad (30)$$

where  $\rho$  is the atoms density,  $\Gamma_e$  is the level radiation width,  $\Gamma$  is the atom level width (including Doppler widening),  $E$  is the electric field strength one can estimate  $d_a$  as

$$d_a = \frac{2\pi \hbar \Gamma^2}{\rho c T \lambda^2 \Gamma_e E} \delta\vartheta \approx 10^{-33} e \quad (31)$$

(here  $\lambda \sim 10^{-4} \text{ cm}$ ,  $E = 10^2 \text{ CGSE}$ ,  $\rho = 10^{17} \text{ atoms/cm}^3$ ,  $\delta\vartheta \approx 10^{-6}$ ,  $T = 1 \text{ hour} = 3,6 \cdot 10^3 \text{ sec}$ ). The similar estimations can be achieved at analysis of birefringence effect.

For comparison it is interesting to note that the best expected EDM measurement limit in recent publications [5] is about  $d_a \approx 10^{-28} e$ , so the advantages of the proposed method becomes evident.

All the said can be applied not only for the optical range but for the radio frequency range as well where the observation of the mentioned phenomenon is also possible by the use of the mentioned atoms and molecules [13].

Thus, we have shown that the T-odd and P-odd phenomena of photon polarization plane rotation and circular dichroism in an electric field are expected to be observable experimentally.

## 5 Conclusion

Analysis of the tensor of dielectric permittivity of medium  $\epsilon_{ik}$  in presence of P- and T-odd interactions results in conclusion of existence of several interesting T-noninvariant optical gyrotropy phenomena. Despite they are small, the use of photon "trap" allow to expect to make possible the experimental observation that let us obtain the important data on the value of T-odd weak interactions.

## References

- [1] Christenson J.H., Cronin J.W., Fitch V.L. and Turlay R. Phys. Rev. Lett. 3 (1964) 1138.
- [2] CPLEAR Collaboration Phys. Lett. B444 (1998) 43.
- [3] Lamoreaux S.K. Nucl. Instrum. Methods. A284 (1989) 43.
- [4] Forte M.J. Journ. Of Phys. G.: Nucl. Part. Phys. G9 (1983) 745.
- [5] Yashchuk V., Budker D., Zolotarev M. Preprint LBNL-42228, August 1998
- [6] Barkov L. M., Melik-Pashayev D., Zolotarev M. Opt. Commun. 70(6), 467-472 (1989).
- [7] Barkov L. M. Zolotarev M. S., Melik-Pashayev D. JETP Letters 48(3), 144-147 (1988).
- [8] Sushkov O. P., Flambaum, V. V., Zh. Eksp. Teor. Fiz. 75(4), 1208-1213 (1978).
- [9] Baranova N.B., Bogdanov Y.V., Zeldovich B.Ya. Uspehi Phiz. Nauk 123, 349 (1977).
- [10] Macaluso D., Corbino O. M. Nuovo Cimento 8, 257 (1898); Ibid. 9, 384-389 (1899).
- [11] Baryshevsky V.G. Phys. Lett A177 (1993) 38.
- [12] Baryshevsky V.G., Baryshevsky D.V. Journ. Of Phys. B: At. Mol. Opt. Phys. 27 (1994) 4421.
- [13] Baryshevsky V.G. J.High Energy Phys. 04 (1998) 018.
- [14] Baryshevsky V.G. LANL e-print archive hep-ph/9912438.
- [15] Landau L., Lifshitz E. Electrodynamics in Continuous Media , 1982, Moscow Science.
- [16] Baryshevsky V.G. LANL e-print archive hep-ph/9912270
- [17] Baryshevsky V.G. Phys. Lett A260 (1999) 24.
- [18] Khriplovich I.B. Parity Nonconservation in Atomic Phenomena. 1991 (London: Gordon and Breach).
- [19] Kozlov M.G., Porsev S.G. Phys. Lett. A142 (1989) 233.
- [20] Landau L., Lifshitz E. Quantum mechanics, 1989, Moscow Science.
- [21] Berestetskii V., Lifshitz E., Pitaevskii L. Quantum electrodynamics, 1989, Moscow Science.

- [22] Meekhof D.H., Vetter P.A., Majumber P.K., Lamoreaux S.K. and Fortson E.N. Phys. Rev. A52 (1995) 1895
- [23] Edwards N.H., Phipp S.J., Baird G. and Nakayama S. Phys. Rev. Lett 74 (1995) 2654
- [24] Cameron R., Cantatore G., Melissinos A.C. et. al. Phys. Rev. D 47 (1993) 3707